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Analysis of numerical integration methods for unmanned aerial vehicle flight dynamics simulation

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ABSTRACT Compute simulation of a UAV flight in a six-degree-of-freedom model involves numerical integration of a system of twelve non-linear ordinary differential equations. The solution quality and simulator speed depend directly on a chosen integration scheme, yet the choice is usually made by habit rather than by quantitative evidence on specific problems. **Purpose:** Provide a quantitative comparison of five one-step integration methods – explicit Euler, Heun, midpoint RK2, classic RK4, and adaptive Runge–Kutta–Fehlberg (RKF45) – on fixed-wing UAV dynamics problems, using the author's Python-based simulator as the implementation platform. **Results:** In a 60-second cruise flight with a step size of 0.01 s, RK4 produces a coordinate error of $8.6 \cdot 10^{-7}$ m, while explicit Euler, at the same step size, accumulates 1.23 m – a six-orders-of-magnitude gap. In a coordinated turn, Euler error reaches 64 m at the same step size, which is unacceptable. The adaptive RKF45 scheme with $\text{tol} = 10^{-6}$ during the cruise reduces the number of right-hand-side evaluations from 24,000 (RK4, $h = 0.01$ s) to 5,900, a fourfold saving at a comparable accuracy. In the wind-gust scenario, the advantage is even larger – 3,700 vs 12,000 evaluations, with both the coordinate and velocity errors smaller than those of fixed-step RK4 by a factor of about twenty. **Practical significance:** The authors give quantitative recommendations for selecting an integrator depending on the flight mode and requirements for determinacy of the computational cycle.

KEYWORDS: fixed-wing unmanned aerial vehicle; flight dynamics; numerical integration; Euler method; Heun method; Runge–Kutta method; adaptive Runge–Kutta–Fehlberg scheme; RKF45; six-degree-of-freedom model; flight simulation

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Научная статья

Анализ численных методов интегрирования при моделировании динамики полета беспилотных летательных аппаратов

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АННОТАЦИЯ Компьютерная симуляция полета беспилотного летательного аппарата в модели с шестью степенями свободы сводится к интегрированию системы из двенадцати нелинейных обыкновенных дифференциальных уравнений. Качество решения и скорость работы симулятора напрямую определяются выбранной схемой интегрирования, однако этот выбор обычно делается по привычке, без количественного обоснования на конкретных задачах. Цель: дать количественное сравнение пяти одношаговых методов интегрирования – явной схемы Эйлера, метода Хойна, Рунге–Кутты второго порядка в midpoint-формулировке, классического RK4 и адаптивной схемы Рунге–Кутты–Фельберга RKF45 – на задачах динамики БПЛА самолетного типа. В качестве программной основы исполь-

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зован разработанный авторами симулятор на Python. Результаты: на крейсерском полете длительностью 60 с при шаге 0,01 с RK4 дает погрешность по координате $8,6 \cdot 10^{-7}$ м, тогда как явная схема Эйлера при том же шаге накапливает ошибку 1,23 м – разница шесть порядков. На координированном развороте ошибка Эйлера при том же шаге достигает 64 м и делает схему непригодной. Адаптивная схема RKF45 с допуском $\text{tol} = 10^{-6}$ на крейсере сокращает число обращений к правой части с 24 тыс. (RK4, $h = 0,01$ с) до 5,9 тыс., т.е. в 4 раза, при сопоставимом порядке точности; на участке с ветровым возмущением преимущество еще заметнее – 3,7 против 12 тысяч обращений, причем ошибка RKF45 оказывается в 20 раз меньше по координате и в 20 раз меньше по скорости, чем у RK4 с фиксированным шагом. Практическая значимость: сформулированы количественные рекомендации по выбору интегратора в зависимости от режима полета и требований к детерминированности вычислительного цикла.

КЛЮЧЕВЫЕ СЛОВА: беспилотный летательный аппарат самолетного типа; динамика полета; численное интегрирование; метод Эйлера; метод Хойна; метод Рунге–Кутты; адаптивная схема Рунге–Кутты–Фельберга; RKF45; шестистепенная модель; симуляция полета

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INTRODUCTION

Fixed-wing unmanned aerial vehicles (UAVs) take a special place in the range of unmanned systems, as the lifting wing provides a significantly greater flight endurance and range compared to multi-rotor aircraft, making this class of aircraft preferable for aerial surveys of extensive areas, mapping, and monitoring of pipelines and power transmission lines. From the selection of an aerodynamic configuration to the adjustment of a stabilization control loop, designing these aircraft without digital simulation is virtually impossible, because flight trials of a full-scale prototype are expensive and involve the inevitable risk of losing the aircraft [1, 2].

A mathematical model for the three-dimensional motion of a fixed-wing UAV is written as a system of twelve non-linear ordinary differential equations (ODEs): six equations for translational and rotational motion in a fixed coordinate system, three kinematic relationships for Euler angles, and three equations for the coordinates of the Earth's centre of mass [1, 3, 4]. Aerodynamic forces and moments enter the right-hand sides as highly non-linear functions of angles of attack, sideslip, Mach number, and control surface deflections. Consequently, the system has no closed analytical solution, and numerical integration is required.

In practice, one of the three approaches is usually used. The explicit Euler scheme remains in use for rapid evaluations and debugging, where the minimum step cost and implementation simplicity are more important than accuracy. The primary work tool is the classic fourth-order Runge–Kutta method (RK4) used to simulate most design and research problems. Where a range of typical process times is large and a single fixed step cannot simultaneously ensure accuracy in fast sections and an acceptable cost in slow sections, embedded pairs – the Runge–Kutta–Fehlberg (RKF45) or Dormand–Prince (DOPRI5) scheme – are used [5–8]. A comparison of these approaches specifically for

fixed-wing UAVs is found in few publicly available quantitative studies. Most frequently, the integrator choice is either made without a discussion or justified by general characteristics of the method without reference to a specific physical problem.

This paper fills in this gap by quantitatively comparing five one-step integration methods based on the criteria of accuracy, computational cost, and suitability for real-time operation on problems typical of fixed-wing UAVs, such as steady-state cruise flight, coordinated turn, and wind gust recovery. The author's Python-based simulation complex is used as the software platform, where all the schemes considered in this work are implemented as successors of a common interface to ensure strict comparability of the results.

FIXED-WING UAV THREE-DIMENSIONAL MOTION MODEL

The motion of a rigid aircraft is described by two vector equations – one for translational motion and the other for rotational motion of the centre of mass – along with kinematic relationships that link angular velocities in a fixed system with derivatives of Euler angles, and equations for converting velocities from the fixed coordinate system to the Earth's system. This produces a system of twelve first-order ODEs, combined into a Cauchy problem (1):

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0, \quad (1)$$

where y is the state vector which includes the components of linear velocity in the fixed coordinate system (u, v, w), the components of angular velocity (p, q, r), the Euler angles (ϕ, θ, ψ), and coordinates of the Earth's centre of mass (x, y, z) (2):

$$y = [u, v, w, p, q, r, \phi, \theta, \psi, x, y, z]^T. \quad (2)$$

The right-hand side $f(t, y)$ includes aerodynamic coefficients of lift, drag, lateral force, and the corresponding moment coefficients which, for a fixed-wing UAV, are assumed to be linear with respect to small deviations in angles α , β and deflections of the elevator (δ_e), ailerons (δ_a), and rudder (δ_r). The magnitude of thrust is proportional to the square of the relative throttle position δ , [2, 3, 9]. In the developed software implementation, the aircraft parameters are encapsulated into the UAVParameters structure (weight, wing span, wing area, principal moments of inertia, and basic aerodynamic derivatives), which enables the reconfiguration of the model for a specific UAV by simply substituting parameters without touching the integrator code.

The flight mode defines the system's nature. In a steady-state level flight, small deviations are described by a linearized model, its eigenvalues split into two groups with different characteristic times: a fast, short-period mode with a period of about one second, and a slow phugoid with a period of tens of seconds in the longitudinal channel, plus a Dutch roll, and the spiral mode in the lateral channel. During manoeuvres – abrupt changes in thrust, entering a tight turn, stall recovery – the non-linear terms in the right-hand side come to the fore, the characteristic times become shorter, and a single fixed step of integration is no longer sufficient to cover the entire range. It is this non-uniformity that determines the requirements for the integrator: a scheme that is convenient for cruising may be insufficiently accurate or stable during manoeuvring [10–12].

NUMERICAL INTEGRATION METHODS CONCERNED

Below, we consider five explicit one-step integration schemes that differ from each other in the number of right-hand side evaluations per step and the order of magnitude of the local truncation error. All of these are implemented in the software suite as successors of the abstract class NumericalIntegrator with a common method ($f; t, y, h$), which ensures that they are directly interchangeable in the main integration loop.

Explicit Euler scheme. This is the simplest one-step method: the status value at each following step is obtained by a single substitution in the right-hand side at a current point (3) [6, 13]:

$$y_{n+1} = y_n + h \cdot f(t_n, y_n). \quad (3)$$

The number of right-hand side evaluations per step is one, which ensures the minimum computational cost. The local truncation error is proportional to h^2 , and the global truncation error is proportional to h ; therefore the scheme is classified as a first-order method. For

long-term simulations of UAV dynamics (ten or more seconds), this accuracy leads to a noticeable accumulation of error, especially when vibrational modes are present. Still, the advantages of the scheme are that the software implementation is trivial and the cost is low, making it a convenient tool for debugging and preliminary evaluation.

Heun's method (improved Euler method). This is a second-order accurate, two-stage explicit scheme. A step involves evaluating two intermediate values in the right-hand side – at the beginning of an interval and at the predicted end obtained by the Euler approximation, – and the resulting increment is then taken as the average of the two evaluations (4) [6, 7]:

$$k_1 = f(t_n, y_n); \quad k_2 = f(t_n + h, y_n + h \cdot k_1), \quad (4a)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2). \quad (4b)$$

The local error is $O(h^3)$, the global error is $O(h^2)$. The cost per step is two right-hand side evaluations. The method is good as an “intermediate” solution where the cost of RK4 is undesirable and the accuracy of Euler is insufficient.

Second-order Runge-Kutta (midpoint). This is an alternative second-order, two-stage explicit scheme which evaluates the right-hand side at the beginning and midpoint of an interval using an intermediate solution computed “halfway” from Euler's extrapolation (5) [7]:

$$k_1 = f(t_n, y_n); \quad k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right); \quad (5)$$

$$y_{n+1} = y_n + h \cdot k_2$$

The per-step cost of this option is the same as for the Heun method: two evaluations in the right-hand side. The numerical experiment on the problems considered (Table 1) shows that both Heun and midpoint are virtually equivalent in terms of ultimate accuracy; and in practical applications, differences between the two are insignificant.

Classic fourth-order Runge-Kutta (RK4). This is a four-stage explicit scheme obtained by matching a Taylor expansion with a set of Simpson quadrature coefficients. A step involves evaluation of the right-hand side at four points: at the beginning, twice at the midpoint with different intermediate state evaluations, and at the end of an interval (6):

$$k_1 = f(t_n, y_n), \quad (6a)$$

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right), \quad (6b)$$

$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right), \quad (6c)$$

$$k_4 = f(t_n + h, y_n + h \cdot k_3). \quad (6d)$$

The resulting increment consists of the weighted sum of four evaluations with the weights of 1:2:2:1 (7):

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4). \quad (7)$$

The local error is of order $O(h^3)$, the global error is of order $O(h^2)$. At the same step h , this reduces the global error approximately $1/h^3$ times compared to the Euler scheme: for $h = 0.01$ s, a theoretical gain is about 10^6 , and in an experiment (please refer to Table 1), the actual Euler/RK4 error ratio on the cruise problem is the same order. The cost per step is four times higher with four right-hand side evaluations instead of one, but the cost is justified by a many times greater accuracy [7, 11, 14].

Adaptive Runge-Kutta scheme (RK4). This is an embedded Runge-Kutta pair of the fourth and fifth orders offered by Erwin Fehlberg in 1969 [15]. Six intermediate slopes $k_1 \dots k_6$ with a fixed set of Fehlberg coefficients are computed in each step, and these estimates k_i are simultaneously reduced to fourth-order (y_4) and fifth-order (y_5) solutions. The difference between them ($y_5 - y_4$) is used as an a posteriori estimate of the local error (8) [7]:

$$err = |y_5 - y_4|. \quad (8)$$

The next step size is adjusted so that the error estimate does not exceed the specified tolerance tol (9):

$$h_{new} = h \cdot s \cdot \left(\frac{tol}{err}\right)^{\frac{1}{5}}, \quad (9)$$

where $s \approx 0.9$ is the empirical safety factor.

The implemented version has additional restrictions: the step should not be reduced more than ten times per iteration and should not be increased more than five times. This prevents both looping on vanishingly small steps in the singularity region and uncontrolled step growth near bifurcation points [16]. The cost per step used is six right-hand side evaluations; if a step is rejected for exceeding the permissible error, the cost for this iteration is reset to zero and the process is repeated with a reduced h .

COMPARATIVE ANALYSIS METHOD

Three test problems encompassing typical flight modes of a fixed-wing UAV were defined for quantitative comparison purposes.

Problem 1: Steady-state cruise flight. A 5 kg UAV with a wingspan of 2 m and wing area of 0.5 m^2 flies at an altitude of 100 m at a velocity of 15 m/s with zero control surface deflection and the thrust mode of 50%. The simulation duration is 60 s. The problem tests the accuracy of integrators in a smooth mode, where the accumulation of error over time is of key significance.

Problem 2: Coordinated turn. The initial conditions are the same, but at the initial moment, the elevator is given a positive deflection $\delta_e = 2^\circ$, and the ailerons are given a positive deflection $\delta_a = 5^\circ$, which leads to a turn with an increasing angle of bank. The

simulation duration is 30 seconds. The problem tests how the integrators behave in the transient mode.

Problem 3: Wind gust recovery. In the cruise mode, a pulse disturbance $\Delta w = 3$ m/s simulating a collision with an updraft is added to the vertical velocity component at $t = 10$ s. The simulation duration is 30 s. The problem tests the stability of a solution on a section with abruptly changing dynamics.

The solution obtained by classic RK4 with a step $h = 5 \cdot 10^{-4}$ s is used as a reference trajectory. At this step, the local truncation error in RK4 is proportional to $h^5 \approx 3 \cdot 10^{-17}$, i.e., by 7–8 orders of magnitude smaller than that yielded by the tested methods at $h = 0.01$ s. This allows the reference to be considered as “accurate” in engineering terms. The error in each tested scheme is evaluated as the maximum Euclidean norm of deviation from the reference separately for the coordinate vector (x, y, z) and the linear velocity vector (u, v, w) across the simulation interval. At the same time, the total number of right-hand side evaluations N_f is recorded, which is a platform-neutral metric to measure computational cost.

NUMERICAL RESULTS OF THE EXPERIMENT

A summary of the metrics measured for Problem 1 at step $h = 0.01$ s is given in Table 1. The columns list the maximum absolute deviation from the reference coordinate and velocity, as well as the total number of right-hand side evaluations over 60 seconds of simulation. The results were obtained using the parameters of a 5 kg UAV with standard aerodynamics (please refer to Fixed-Wing UAV Three-Dimensional Motion Model section), under the initial conditions of cruise equilibrium at an altitude of 100 m and a velocity of 15 m/s.

Table 1
Comparison of methods on a cruise problem ($h = 0.01$ s, $T = 60$ s)

Method	Coordinate error, m	Velocity error, m/s	N_f
Euler	$1.23 \cdot 10^0$	$2.88 \cdot 10^{-2}$	6001
Heun	$4.89 \cdot 10^{-4}$	$8.26 \cdot 10^{-4}$	12 002
RK2 (midpoint)	$8.87 \cdot 10^{-4}$	$8.25 \cdot 10^{-4}$	12 002
RK4	$8.60 \cdot 10^{-7}$	$3.95 \cdot 10^{-7}$	24 004
RKF45 (tol = 10^{-6})	$7.48 \cdot 10^{-6}$	$7.27 \cdot 10^{-7}$	5904

The situation is expectable in terms of orders of magnitude, but the numerical relations deserve further investigation. Switching from Euler to second-order schemes reduces the coordinate error from 1.23 m to $4.89 \cdot 10^{-4}$ m – approximately 2,500 times – while doubling the step cost. This is the most beneficial transition in terms of cost/accuracy. Heun and RK2 midpoint are virtually equivalent in both metrics, with

discrepancies being within the rounding error range. RK4 with the same step size of 0.01 s yields a coordinate error of $8.6 \cdot 10^{-7}$ m – three orders of magnitude better than the second-order schemes and almost indistinguishable from the reference. The behaviour of RKF45 is worth a separate note: just 808 steps used versus 6,000 in the fixed-order schemes, the average step size used is approximately 0.074 s (seven times larger than in the fixed-order schemes), and the total number of right-hand side evaluations N_f is 5,904, which is 75% fewer than in the case of RK4 (24,004 evaluations) and comparable to the cost in Euler (6,001).

The situation changes in Problem 2 (turn). The Euler integrator with a step size of 0.01 s accumulates a 64-metre coordinate deviation in 30 seconds, which is effectively a failure, as with this level of error, no turn is possible. The second-order schemes return to a range of tens of centimetres (0.68 metre coordinate errors for both Heun and RK2). RK4 yields a coordinate error of $2.05 \cdot 10^{-4}$ m and a velocity error of $2.57 \cdot 10^{-6}$ m/s, which is three orders of magnitude better than in the second solution. RKF45 involves 9,258 right-hand side evaluations and yields $2.37 \cdot 10^{-5}$ m and $3.65 \cdot 10^{-6}$ m/s errors, saving about 23 per cent compared to RK4 with $h = 0.01$ s (12,000 evaluations), with the coordinate error being ten times smaller. Compared to RK4 with $h = 0.005$ s (24,004 evaluations), the adaptive scheme saves 61% in evaluations while providing a comparable level of accuracy.

Problem 3 (wind gust) is most illustrative in demonstrating the benefit of step adaptation. At the moment of a pulse disturbance, the vertical component of velocity changes by 3 m/s, and the aircraft enters a transient process with sharply accelerated dynamics. RKF45 responds to the growth of the local error by decreasing the step in this particular interval; in 30 seconds of simulation, the adaptive scheme uses 509 steps versus 3,000 in the fixed methods, with the average step being about 0.059 s. In terms of accuracy, RKF45 considerably outperforms all the other options with a coordinate error of $1.38 \cdot 10^{-4}$ m and a velocity error of $5.81 \cdot 10^{-3}$ m/s, which is 20 times more accurate than in RK4 with $h = 0.01$ s for both metrics, and better than in RK4 with $h = 0.005$ s ($1.23 \cdot 10^{-3}$ m and $5.22 \cdot 10^{-2}$ m/s, respectively). The explanation is plain: a fixed step of 0.01 s in the pulse section is too large to track a rapid response, while the adaptive scheme drops significantly lower in this interval, without carrying an excessive cost in smooth sections before and after the disturbance. The Euler integrator maintains stability, but yields a 1-metre coordinate error and a 0.7-m/s velocity error; the second-order schemes yield a velocity error of 0.06 to 0.36 m/s, which is already unacceptable for the quantitative evaluation of the transient process.

The step stability margin was tested separately. In 30 seconds of simulation in the cruising mode, the Euler method yields an error of approximately 1 m at

$h = 0.01$ s; 2 m at $h = 0.02$ s; and a register overflow (NaN) at $h = 0.05$ s, meaning that the scheme loses stability. RK4 maintains stability at h of up to 0.05 s (an error of $6 \cdot 10^{-4}$ m) and also goes to NaN at $h = 0.08$ s. A working range for RK4 step in the problem concerned is approximately [0.005 s; 0.05 s]. RKF45 is insensitive to the initial step size: with an initial h of 10^{-4} s to 10^{-1} s, the final result remains correct, since the very first iteration adjusts the step size to the local dynamics.

RECOMMENDATIONS FOR CHOOSING AN INTEGRATION METHOD

To summarize the results of the experiment, we can formulate practical recommendations applicable to modelling problems for fixed-wing UAVs.

In the preliminary model evaluation and debugging phase, when the problem structure is still being specified, it is acceptable to use the explicit Euler scheme with a step size of about 10^{-3} to 10^{-2} s. Its trivial implementation reduces the risk of errors and facilitates the interpretation of failures; however, it cannot be used for meaningful calculations over long horizons: as it was shown above, the error during the turn quickly becomes unacceptable.

The main working tool is the classic RK4 method with the step size of about 10^{-2} s. In the modes considered, it yields a coordinate error of 10^{-7} m (cruiser) to 10^{-3} m (gust) with a predictable number of right-hand side evaluations, which is convenient for both offline calculations and on-board systems with real-time requirements.

Modes with rapidly changing dynamics (spins, stall recovery, pulse wind disturbances, hard landing) are best integrated using the adaptive RKF45 algorithm with tolerance $tol = 10^{-6}$. In the test wind gust problem, this integrator yielded an error that is an order of magnitude smaller than that of RK4 while using three times fewer right-hand side evaluations. In case of advanced problems, it is reasonable to replace RKF45 with DOPRI5 or DOPRI87 with better asymptotic error constants [17].

Where the computational cycle is to be completed within a strictly determined timeframe, which is a typical case for on-board flight controllers, the use of the adaptive methods is impossible. Unpredictable step selection time disturbs the guaranteed scheduling in the operating system. In this case, a fixed step in RK4 selected with a margin for the worst-case scenario can be used [18, 19].

CONCLUSION

The paper provides a quantitative comparison of the five one-step integration schemes – the explicit

Euler scheme, Heun's method, midpoint RK2, classic RK4, and the adaptive Runge–Kutta–Fehlberg (RKF45) method – with respect to problems for modelling dynamics of a fixed-wing UAV. The comparison was made for three standard modes: steady-state cruise flight, coordinated turn, and wind gust recovery. The classic solution by the RK4 method with the step size of $5 \cdot 10^{-4}$ s was used as a reference; the error was measured as the maximum Euclidean norm of deviation from the reference state in coordinate and velocity.

The measured data (Table 1 and the Numerical Results of the Experiment section) are consistent with theoretical estimates of orders and provide a well-defined basis for practical recommendations. RK4 with a step size of 0.01 s yields coordinate error from $8.6 \cdot 10^{-7}$ m in the cruise flight to $2.9 \cdot 10^{-3}$ m for a gust problem under a predictable computational load, making it a reasonable default choice for research calculations. The adaptive RKF45 method with $tol = 10^{-6}$ saves 23% to 75% in terms of the number of right-hand side evaluations compared to RK4 with a fixed step size of 0.01 s and from 61% to 88% compared to RK4 with $h = 0.005$ s, and is more accurate in areas with heterogeneous dynamics: in the wind gust problem, the RKF45 error is 20 times smaller than that of RK4

with the same average step size. The explicit Euler scheme accumulates 64 m of coordinate errors in the turn problem and is unsuitable for meaningful calculations. The second-order methods (Heun and RK2) are close to each other in terms of overall accuracy. These are inferior to RK4 by 3–3.5 orders of magnitude in coordinates for the cruise and turn problems, and by less than one order of magnitude for the gust problem, since in both cases, the main error accumulates during the pulse phase. The use of the second-order methods is justified in narrow scenarios where a computational budget is strictly limited and simulation durations are short.

The next step will be to extend the comparison to implicit schemes (Rosenbrock, BDF), which are necessary when the UAV model is supplemented with a description of a fast electric propulsion sub-system and the ODE system becomes rigid. Of practical interest is also the automatic switching of the integrator based on the mode change detector: fixed RK4 during normal mission phases and RKF45 in abnormal ones. This hybrid solution preserves the determinacy of the main cycle while simultaneously employing adaptivity where it is truly needed.

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