

Original article

UDC 656.2

doi: 10.46684/2023.2.3

## On some mechanical characteristics of the ballast in assessing the stress-strain behaviour of railway tracks

Vladimir I. Smirnov<sup>1✉</sup>, Sergey A. Vidyushenkov<sup>2</sup>

<sup>1,2</sup> Emperor Alexander I St. Petersburg State Transport University (PGUPS); St. Petersburg, Russian Federation

<sup>1</sup> vsmirnov1@gmail.com✉; <https://orcid.org/0000-0001-5345-6165>

<sup>2</sup> baklava@mail.ru; <https://orcid.org/0000-0002-5734-810X>

**ABSTRACT** On the basis of recent research, a review has been carried out on the experimental determination of the mechanical parameters of the materials used as ballast material. Considerable attention is given to the well-founded selection of input data for the calculation, i.e. quantitative characteristics of the elastic properties of the materials used to form the ballast layer, which is treated as a continuous medium. It appears that this approach makes it possible to assess correctly the influence of grain (grain size distribution) distribution, ballast layer thickness and material type on the stability of a railway track to vertical and horizontal disturbances. The data of this review show that material properties and particle size have a significant impact on elastic moduli and in finite-element modelling of static strength problems, grain-size distribution and material properties are only taken into account through these moduli. Experimental results show a non-linear dependence of the elastic moduli on the stress behaviour of the ballast prism, which is related to a densification of the medium in compression. However, it is established that, after a sufficiently large number of loading cycles, the medium can be treated as linearly elastic. In general, the results of the research allow establishing requirements for damping, geometric and granulometric parameters of the ballast prism.

**KEYWORDS:** ballast layer; ballast materials; grain size distribution; compression; linear-elastic medium

**For citation:** Smirnov V.I., Vidyushenkov S.A. On some mechanical characteristics of the ballast in assessing the stress-strain behaviour of railway tracks. *BRICS transport*. 2023; 2(2):3. <https://doi.org/10.46684/2023.2.3>.

Научная статья

## О некоторых механических характеристиках балласта при оценке напряженно-деформированного состояния железнодорожного пути

В.И. Смирнов<sup>1✉</sup>, С.А. Видюшенков<sup>2</sup>

<sup>1,2</sup> Петербургский государственный университет путей сообщения Императора Александра I (ПГУПС); г. Санкт-Петербург, Россия

<sup>1</sup> vsmirnov1@gmail.com✉; <https://orcid.org/0000-0001-5345-6165>

<sup>2</sup> baklava@mail.ru; <https://orcid.org/0000-0002-5734-810X>

**АННОТАЦИЯ** На основании зарубежных публикаций проведен обзор по экспериментальному определению механических параметров материалов, используемых в качестве балластного материала. Данные этого обзора показывают, что свойства исходного материала и размеры частиц оказывают влияние на упругие модули и при конечно-элементном моделировании статических задач гранулометрический состав и свойства материала учитываются лишь через модули. Экспериментальные результаты показывают нелинейную зависимость модулей от напряженного состояния, что связано с уплотнением среды при сжатии. Однако указывается, что после достаточно большого числа циклов нагружения можно считать среду линейно-упругой.

© V.I. Smirnov, S.A. Vidyushenkov, 2023

**КЛЮЧЕВЫЕ СЛОВА:** балластный слой; балластные материалы; гранулометрический состав; сжатие; линейно-упругая среда

**Для цитирования:** Смирнов В.И., Видюшенков С.А. О некоторых механических характеристиках балласта при оценке напряженно-деформированного состояния железнодорожного пути // Транспорт БРИКС. 2023. Т. 2. Вып. 2. Ст. 3. <https://doi.org/10.46684/2023.2.3>.

## INTRODUCTION

The ballast layer, as an element of the track structure, significantly predetermines durability of the track as a whole, serviceability of the subgrade and scope of maintenance and repair works.

Existing semi-empirical path calculation methods give a linear dependence of deformation (deflection) on the speed of the vehicle in a fixed section. However, experiments show that this dependence is valid only at low speeds (up to 100–150 km/h). At higher speeds this dependence becomes non-linear and gives a significant divergence from theory.

The interaction of the track and rolling stock at different speeds is determined by force interaction between wheel and rail, in which a great role is played by static and dynamic properties of the ballast prism. In general, the calculation of this interaction is related to the solution of a dynamic problem in elasticity theory. In fact, one should consider the oscillation of an elastic volume under the influence of external forceful disturbances. Taking into account wave processes in such systems as “vehicle-track” is a difficulty, which, on the one hand, is related to the correct setting of appropriate boundary conditions, on the other hand, with a reasonable choice of physical and mechanical properties of the ballast material, which are put in the mechanical-mathematical model of the deformation of the track.

## MATERIALS AND METHODS

The ballast for the upper part of the track is a granular material designed to distribute the load applied by the moving train evenly. The ballast consists of particle elements between 20 and 60 mm in size. Today it is generally accepted that high quality ballast must have well defined particles, high density, high shear strength, high rigidity, high resistance to abrasion, uneven surface and minimum number of cracks and inclusions [1–4]. Many properties of the ballast will change under dynamic loading due to ballast discolouration, deformation and contamination. Ballast contamination causes reduction in drainage capacity, hydraulic erosion, reduced stability due to sliding particles, wear

on the bottom of the track and track damage due to increased water pressure (during rain) in the pores between the particles in the ballast.

This raises the need to improve drainage and strength properties, which are in a sense contradictory. It is first necessary to consider physical and mechanical properties of the ballast and how to mechanically and mathematically model the behaviour of the railway track.

The simplest model of track is a sleeper as a bar on an elastic base under the impact of a concentrated load acting from the side of the wheel. The resilient bed must take account of the stiffness of the sleepers, ballast and track bottom (earth, etc.). The values for the elasticity modulus of the rail support are derived from the results of field tests of the track sections under operating loads. It is defined as the specific force (acting per unit of rail length) per unit of track deflection

$$U = \frac{1}{4} \left( \left( \frac{P}{\delta} \right)^4 \frac{1}{EI} \right)^{1/3}, \quad (1)$$

where  $P$  is acting load (force);  $\delta$  is deflection of the rail under the point of load application;  $E$  is Young modulus of rail steel;  $I$  is moment of inertia of rail cross-section.

More complex analytical track models represent rails and sleepers as bars resting on a multi-layer base that includes a ballast layer and soil. These models include computer programmes such as ILITRACK (1975), MULTA (1978), GE-OTRACK (1980, 2000), KENTRACK (1986), RAIL (2004). Finite element models have also been developed based on one-, two- and three-dimensional representations of track elements. In the last fifteen years, calculation programmes based on combined boundary and finite element methods have been widely used [5–8].

There are two main problems associated with the physical and mechanical properties of ballast:

- determination of maximum elastic deformations and deflections caused by wheel loads;
- accumulation of plastic deformations as a result of repeated characteristic loads.

The concept of elastic recovery modulus (unloading-loading modulus)  $E_{ur}$  is used to describe the behaviour of the ballast under repetitive loading conditions. The elastic recovery modulus is defined as the ra-

tio of the deviator of repetitive stresses to the restored part of the axial strain [9]. The most commonly used expression for this modulus is

$$E_{ur} = K p_r \left( \frac{\sigma_3}{p_r} \right)^n, \quad (2)$$

where  $K$ ,  $n$  are material constants determined by tests;  $p_r$  is reference pressure;  $\sigma_3$  is primary stress.

In [10], on the basis of in-situ experiments, the authors state that the elastic recovery modulus of the ballast layer under given loading conditions is one of the most important factors influencing the value of stress deviation at the soil-ballast boundary. With a minimum ballast layer thickness of 0.3 m, the stress-deviation stresses are maximal at the end of the sleeper and minimal at the centre of the sleeper. An increase in ballast thickness from 140 to 550 MPa results in a 35 % decrease in the stress-deviation stress at the ground near the end of the sleeper.

It was also established that the elastic recovery modulus has the greatest influence on the deflection of sleepers under the wheels. GEOTRACK software was used to determine the average deformation of the ballast in the vertical direction. The average deformation was calculated as the ratio of the difference in displacement of the upper and lower surfaces of the ballast to the initial thickness of its layer. If  $E_{ur}$  is reduced from 689 to 55 MPa, the deformation of the ballast is increased by a factor of 9. Based on the results of these studies, the modulus of elasticity of the base  $U(1)$  was estimated to increase by 20 % with an increase of  $E_{ur}$  within the specified limits.

In order to investigate the residual deformation in ballast, three-axial compression tests must be carried out on special stabilometers under repeated loading. The tests carried out by many researchers in the last 30 years showed that there is a limit for the ratio of stress deviator of cyclic failure to the value of stress deviator of static failure

$$K_c = \frac{(\sigma_1 - \sigma_3)_{cf}}{(\sigma_3 - \sigma_3)_f},$$

where  $\sigma_1$ ,  $\sigma_3$  are primary stresses.

If the cyclic stresses are less than  $K_c(\sigma_1 - \sigma_2)_f$ , the residual strain of the ballast test piece converges to a constant value for that material. The ballast test piece then behaves as a quasi-elastic material. If the above-mentioned limit is exceeded, the cyclic and plastic strain is increased almost linearly from cycle to cycle and the test piece fails within a short time [11]. Different data are given in the literature regarding the value. Thus,  $K_c = 0.8$  according to the results obtained in [11] and  $K_c = 0.6$  according to the publication [12].

Several expressions have been proposed for the estimation of permanent strain in ballast caused by repeated loading.

The Office of Research and Testing of the International Union of Railways (ORE of IUR) suggests using the formula [13]

$$\varepsilon_N = 0.082(100n - 38.2)(\sigma_1 - \sigma_3)^2(1 + 0.2 \log N), \quad (3)$$

where  $\varepsilon_N$  is plastic strain after  $N$  cycles of loading;  $n$  is initial porosity of the test piece;  $\sigma_1 - \sigma_3$  is stress deviator.

In paper [14] it is suggested to determine  $\varepsilon_N$  as follows

$$\varepsilon_N = \varepsilon_1(1 + C \log N), \quad (4)$$

where  $C$  is constant of the material,  $C = 0.2-0.4$ .

Equations (3) and (4) show that plastic strain in any cycle  $\varepsilon_N$  can be calculated as a function of the number of applied cycles  $N$  or of the plastic strain after the first loading cycle  $\varepsilon_1$ , irrespective of the state of stress and degree of compaction of the test piece. The value  $\varepsilon_1$  can be determined by the formula

$$\varepsilon_1 = \varepsilon_a - \varepsilon_{ur}, \quad (5)$$

where  $\varepsilon_a$  is axial strain from the stress deviator  $\sigma_1 - \sigma_3$ ;  $\varepsilon_{ur}$  is recoverable deformation during unloading. Deformation  $\varepsilon_a$  is determined from the ratio

$$\varepsilon_a = \frac{(\sigma_1 - \sigma_3) / E_i}{1 - \frac{(\sigma_1 - \sigma_3)(1 - \sin \varphi) R_f}{2(c \cos \varphi + \sigma_3 \sin \varphi)}},$$

$$(\sigma_1 - \sigma_3)_f = R_f(\sigma_1 - \sigma_3)_{ult},$$

where  $E_i$  is initial tangential elasticity modulus of the ballast;  $\varphi$  is internal friction angle;  $R_f$  is the ultimate fracture ratio, is always less than 1 and is largely independent of  $\sigma_3$ ;  $c$  is coupling;  $(\sigma_1 - \sigma_3)_f$  is compressive strength (fracture stress difference obtained in triaxial tests);  $(\sigma_1 - \sigma_3)_{ult}$  is asymptotic value of the stress deviation derived from the test results.

The level of stresses in the ballast layer depends not only on the magnitude of the applied loads but also on the resistance to them from sleepers, fasteners and ballast layer parameters. Numerous field experiments have demonstrated that the speed of a train significantly affects the magnitude of the stresses exerted in the ballast. The maximum vertical dynamic stresses in Hannover–Würzburg line were measured [11]. When the speed of a train is increased from 150 km/h to 300 km/h, the maximum vertical stresses were 70 and 100 kPa, respectively. The authors [11, 14] defined the dynamic factor as the ratio of the dynamic stress in the ballast to the static stress. Its value was equal to 1.7 when the speed changed from 150 to 300 km/h. It has been reported that stresses on the sleeper-ballast soil interface for heavy-load trains can reach a value of 450 kPa at a speed of 170 km/h [11].

The horizontal stresses have been measured [15] and are not as significant as the vertical stresses. The maximum value is 90 kPa at a speed of 150 km/h and a vertical load of 140 kN.

The value of the elastic modulus in the full compression test can be obtained from the expression for generalized Hooke's law

$$E = \frac{\partial \sigma_a}{\partial \varepsilon_a} - 2\nu \frac{\partial \sigma_3}{\partial \varepsilon_a}. \quad (6)$$

where  $\partial \sigma_a$ ,  $\partial \varepsilon_a$  is change in axial stress and strain respectively;  $\partial \sigma_3$  is change of volumetric stress;  $\nu$  is Poisson ratio.

Since in full-compression tests there is no change in the volumetric stress and the measurement data is recorded at discrete points in time, formula (6) takes the form of

$$E = \frac{\Delta \sigma_a}{\Delta \varepsilon_a}. \quad (7)$$

Since soil and ballast do not behave linearly under loading and unloading, several possible ways of determining the modulus of elasticity of ballast can be suggested. The following interpretations of "elastic moduli" are currently known:

1. The initial tangential elastic modulus, defined as the initial slope of the stress-strain curve [16].

2. The modulus of loading-unloading (recovery) is defined as the ratio of the stress deviator of all-round compression to the corresponding strain at the end of the loading-unloading cycle [17]. This modulus was originally used to evaluate the elastic behaviour of soils under cyclic loading.

3. Secular modulus, defined as the ratio of half of the maximum stress deviator to the corresponding strain [18].

All of the above moduli can be determined by statistical processing of the experimental data and are represented in the form of the formula

$$E/p_r = K(\sigma_3/p_r)^n, \quad (8)$$

where  $p_r$  is the reference pressure, usually taken as 1 kPa. To find  $K$  and  $n$  from relation (8) we obtain

$$\lg(E/p_r) = \lg K + n \lg(\sigma_3/p_r). \quad (9)$$

The linear regression analysis is then applied to the experimental data and the dependence of  $E/p_r$  on  $\sigma_3/p_r$  is plotted, from where the  $K$  and  $n$  coefficients are found. The dependence of these values on bulk stresses, particle size and ballast material can be investigated.

The values of elastic moduli for different materials may have different dependencies on the particle size of the ballast. It is possible to trace such dependencies

for limestone, basalt and granite from research material [19–21]. Due to limited and often inconsistent experimental data, further tests are required in order to draw definitive conclusions. Ballast used in railways consists of different particle sizes. GOSTs and national regulations regulate the use of a particular type and size of particles.

### Granite ballast

In [20] data on tests of dry granite ballast particles of two sizes as well as mixtures of these particles are given:

a) fine particles (20 mm diameter) by volume as much as the larger fraction (50 mm) and the fine particles were placed on top;

b) the fines placed on top have a volume equal to 1/3 of the volume of the coarse fraction;

c) the fines placed on top are 2/3 the volume of the coarse fraction;

d) the fines placed on top were 1 and 2 layers.

The tests were carried out under cyclic loading until 105 cycles were reached. From the test results, the authors concluded that the values  $E_{ur}$  and  $E_i$  for all particle sizes increased as the volumetric stress increased during the test. Particles with a small cross-sectional area and lower porosity had higher modulus value  $E_i$  than larger particles. Modulus values  $E_{ur}$  are increased with increasing number of load cycles for fine and coarse ballast. The same tendency was observed for ballast types (a) to (d) above. The modulus values  $E_{ur}$  for cases (a) to (c) were about 30 % higher than for ballast of only coarse particles. The values of  $E_{ur}$  for case (d) were roughly the same as for ballast of only coarse particles at pressures of 40 and 90 kPa.

The results are summarised in Table 1. The conclusion to be drawn from this study is that a ballast prism consisting of two layers of different particle sizes has a higher shear strength and load-displacement modulus than a ballast of the same particle size.

### Limestone ballast

Estimates of elastic moduli from triaxial tests of limestone ballast are given in [19]. The results were obtained on test pieces of five sizes. Test pieces L-2.36 consisted of 2.39–4.75 mm ballast particles; L-4.75 had ballast particle sizes of 4.75–9.5 mm and L-9.5 had ballast particle sizes of 9.75–19 mm. The following formulas can be proposed to determine the parameters required for the estimation of the modulus  $E_{ur}$  as a function of the particle size

$$n = 0.5624D^{-0.1655}, \quad R = 0.403;$$

$$\lg K = 4.102D^{0.0464}, \quad R = 0.619,$$

Table 1

Strain and initial modulus values of the granite ballast

| Average diameter<br>particle diameter, mm | Pressure, kPa | $E_i$ , MPa | $E_{ur}$ , MPa<br>10 <sup>2</sup> cycles | $E_{ur}$ , MPa<br>10 <sup>3</sup> cycles | $E_{ur}$ , MPa<br>10 <sup>4</sup> cycles | $E_{ur}$ , MPa<br>10 <sup>5</sup> cycles |
|-------------------------------------------|---------------|-------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| 50                                        | 40            | 55          | 208                                      | 244                                      | 290                                      | 292                                      |
| 50                                        | 90            | 55          | 292                                      | 323                                      | 358                                      | 368                                      |
| 50                                        | 140           | 81          | 354                                      | 377                                      | 388                                      | 401                                      |
| 20                                        | 40            | 33          | 302                                      | 388                                      | 489                                      | 529                                      |
| 20                                        | 140           | 228         | 563                                      | 672                                      | 737                                      | 818                                      |
| 20                                        | 240           | –           | 1327                                     | 1533                                     | 1596                                     | 1949                                     |
| 20 (1/3 of the volume)                    | 40            | 72          | 251                                      | 298                                      | 305                                      | 370                                      |
| 20 (1/2 of the volume)                    | 40            | 71          | 280                                      | 303                                      | 364                                      | 376                                      |
| 20 (1/2 of the volume)                    | 90            | –           | 368                                      | 411                                      | 466                                      | 493                                      |
| 20 (2/3 of the volume)                    | 40            | 44          | 234                                      | 297                                      | 293                                      | 394                                      |
| 20 (1 layer)                              | 40            | 36          | 207                                      | 219                                      | 267                                      | 287                                      |
| 20 (2 layers)                             | 40            | 46          | 186                                      | 236                                      | 256                                      | 299                                      |
| 20 (1 layer)                              | 90            | –           | 282                                      | 300                                      | 321                                      | 345                                      |

where  $R$  is coefficient of determination;  $D$  — average particle size. Note that the values of  $n$  obtained in the tests are higher than those predicted by Hertz contact theory ( $n = 0.333$ ).

To estimate modulus  $E_i$  of dry ballast depending on particle size the following values were obtained

$$n = 0.6006D^{-0.021}, \quad R = 0.938;$$

$$\lg K = 3.5065D^{0.0179}, \quad R = 0.708.$$

The following parameter values are proposed for determining the  $E_{50}$  secant modulus

$$n = 0.7045D^{-0.1633}, \quad R = 0.532;$$

$$\lg K = 3.2719D^{0.0611}, \quad R = 0.655.$$

These parameter values are close to those proposed in [18], where  $n = 0.5$  and  $\lg K = 3.61$  are valid for almost all ballast particle sizes.

### Basalt ballast

The values of elastic moduli of basalt ballast obtained experimentally in [19] are given in Table 2. The dimensions are the same as for limestone at the same value of volumetric compression pressure.

Dependence of the initial tangential modulus parameter  $E_i$  of dry ballast on particle size

$$n = 0.381D^{0.0435}, \quad R = 0.403;$$

$$\lg K = 0.0057D + 4.0099, \quad R = 0.771.$$

Similar parameters for the secant module  $E_{50}$

$$n = 0.9647D^{-0.473}, \quad R = 0.835;$$

$$\lg K = 0.4505D + 3.0307, \quad R = 0.857.$$

Parameters of module  $E_{ur}$  of the dry ballast

$$n = 0.1794D^{0.2869}, \quad R = 0.964;$$

$$\lg K = 4.763D^{-0.0241}, \quad R = 0.924.$$

Table 2

Elastic modulus values of basalt ballast

| Dimension type | Pressure<br>$\sigma_3$ , kPa | Initial<br>modulus $E_i$ ,<br>kPa | Secant<br>modulus $E_{50}$ ,<br>kPa | Unloading-<br>loading module<br>$E_{ur}$ , kPa |
|----------------|------------------------------|-----------------------------------|-------------------------------------|------------------------------------------------|
| B-2,36         | 35                           | $4.58 \times 10^4$                | $2.71 \times 10^4$                  | $1.04 \times 10^5$                             |
|                | 70                           | $5.76 \times 10^4$                | $4.24 \times 10^4$                  | $1.20 \times 10^5$                             |
|                | 105                          | $7.32 \times 10^4$                | $4.57 \times 10^4$                  | $1.39 \times 10^5$                             |
| B-4,75         | 35                           | $4.72 \times 10^4$                | $2.44 \times 10^4$                  | $1.06 \times 10^5$                             |
|                | 70                           | $5.49 \times 10^4$                | $3.71 \times 10^4$                  | $1.28 \times 10^5$                             |
|                | 105                          | $7.33 \times 10^4$                | $4.25 \times 10^4$                  | $1.45 \times 10^5$                             |
| B-9,5          | 35                           | $5.76 \times 10^4$                | $5.11 \times 10^4$                  | $1.10 \times 10^5$                             |
|                | 70                           | $7.59 \times 10^4$                | $5.92 \times 10^4$                  | $1.62 \times 10^5$                             |
|                | 105                          | $9.42 \times 10^4$                | $6.81 \times 10^4$                  | $1.64 \times 10^5$                             |

Data for basalt in [19] and [21] differ from each other with the number of load cycles 102 but the difference is insignificant. Possible divergences are explained by different origin of the material and different types of its processing as well as by the fact that particle sizes in [21] ranged from 16 to 53 mm. Therefore, a mixture of pre-compacted particles with different sizes was tested. The data of [21] are based on a higher number of loading cycles than in [19] and therefore have a significance of their own. Before the values of elastic moduli can be used in practical calculations, ballast test pieces must be tested in facilities which allow for the triaxial stress state.

The tests [19] of the pieces exposed to water at pressures of 35, 70 and 105 kPa showed that the values of elastic moduli of dry test pieces are 25 % less than the corresponding values of wet test pieces.

## CONCLUSION

The values of elastic initial modulus and secant modulus of ballast of basalt are larger for ballast particle sizes of 4.75–60 mm than for ballast of limestone. The difference can be regarded as insignificant. The modulus values increase with ballast particle enlargement in the above size range.

The loading-unloading modulus of limestone is slightly higher than that of basalt. Both materials have similar moduli values when the same particle size is used. This conclusion is based on tests up to  $5 \cdot 10^4$  load cycles. Additional research with  $5 \cdot 10^5$  loading cycles is necessary for final conclusions.

Increase of ballast particles size in above range of values results in corresponding increase of elastic moduli.

The above expressions for initial and loading-unloading modulus values depending on ballast particle size are valid within 50 % confidence limits and for the estimation of final modulus within 100 % confidence limits.

The values of elastic moduli of dry ballast are on average 25 % lower than those of wet ballast.

Double ballast of different particle sizes has higher shear resistances and load-displacement modulus values than single-layer ballast of the same particle size.

There are limited databases available in the public domain on research into the elastic moduli of ballast of different materials in relation to particle size. There are differences between domestic test rules and GOSTs and foreign ones. Therefore, the above moduli values require experimental confirmation by testing the type of ballast which is to be used in practice for track construction.

## REFERENCES/ЛИТЕРАТУРА

1. Chrismer S.M. Consideration of factors affecting ballast performance. *AREA, Bulletin 704 AAR Research and Test Department Report N. WP-110*. 1985;118-150.
2. Jeffs T., Marich S. Ballast characteristics in the laboratory. *Proceedings of a Conference on Railway Engineering*. 1987;141-147.
3. Indraratna B., Ionescu D., Christie D. State-of-the-art large scale testing of ballast. *Proceedings of CORE2000. Railway Technology for the 21st Century, Conference on Railway Engineering*. 2000;24.1-24.13.
4. Esweld C. *Modern Railway track*. 2nd edn. Zaltbommel, MRT Productions, 2001;632.
5. Jacobson L. User element for ABAQUS designed to represent ballast resistance. *SP Technical Notes: 12, Swedish National Testing and Research Institute*. 2005;9.
6. Chang C.S., Adegoke C.W., Selig E.T. GEOTRACK Model for Railroad Truck Performance. *Journal of the Geotechnical Engineering Division*. 1980;106(11):1201-1218. DOI: 10.1061/ajgeb6.0001059
7. Mohammadi M., Karabalis D.L. Dynamic 3-D soil-railway track interaction by BEM-FEM. *Earthquake Engineering & Structural Dynamics*. 1995;24(9):1177-1193. DOI: 10.1002/eqe.4290240902
8. Adam M., Pflanz G., Schmid G. Two- and three-dimensional modelling of half-space and train-track embankment under dynamic loading. *Soil Dynamics and Earthquake Engineering*. 2000;19(8):559-573. DOI: 10.1016/s0267-7261(00)00068-3
9. Monismith C.L., Seed H.B., Mitry F.G., Chan C.K. Predictions of pavement deflections from laboratory tests. *Second International Conference on the Structural Design of Asphalt Pavements*. 1967;53-88.
10. Li D., Selig E.T. Method for Railroad Track Foundation Design. I: Development. *Journal of Geotechnical and Geoenvironmental Engineering*. 1998;124(4):316-322. DOI: 10.1061/(asce)1090-0241(1998)124:4(316)
11. Kempfert H.G., Vogel W. Grundsatzfragen des Erd- und Grundbaus für Hochgeschwindigkeits-strecken. *AET Archiv für Eisenbahntechnik, 44, Ingenieurbauwerke der Neubaustrecken der Deutsche Bundesbahn*. 1992;11-38.
12. Raymond G.P. Design for railroad ballast and subgrade support. *Journal of the Geotechnical Engineering Division*. 1978;104(1):45-60. DOI: 10.1061/ajgeb6.0000576
13. Selig E.T., Waters J.M. *Track Geotechnology and Substructure Management*. Thomas Telford, 1994. DOI: 10.1680/tgasm.20139
14. Alva-Hurtado J.E., Selig E.T. Permanent strain behavior of railroad ballast. *Proceedings of The International Conference on Soil Mechanics and Foundation Engineering, 10th*. 1981;1:543-546.

15. Raymond G.P., Davies J.R. Triaxial tests on dolomite railroad ballast. *Journal of the Geotechnical Engineering Division*. 1978;104(6):737-751. DOI: 10.1061/ajgeb6.0000646
16. Lambe T.W., Whitman R.V. *Soil Mechanics, SI Version*. John Wiley and Sons, 1979;145-159.
17. Janardhanam R., Desai C.S. Three-Dimensional Testing and Modeling of Ballast. *Journal of Geotechnical Engineering*. 1983;109(6):783-796. DOI: 10.1061/(asce)0733-9410(1983)109:6(783)
18. Vermeer P.A., Schanz T. Evaluation of soil stiffness parameters. *ASCE Workshop on Computational Geotechnics*. University of Colorado, Boulder. 1998;5.
19. Kaya M., Jernigan R., Runesson K., Sture S. *Reproducibility and conventional triaxial tests on ballast material*. Technical Report, Department of Civil Environmental and Architectural Engineering, University of Colorado at Boulder, Colorado, USA, 1997;1:43; 2:40.
20. Anderson W.F., Fair P. Behavior of Railroad Ballast under Monotonic and Cyclic Loading. *Journal of Geotechnical and Geoenvironmental Engineering*. 2008;134(3):316-327. DOI: 10.1061/(asce)1090-0241(2008)134:3(316)
21. Indraratna B., Ionescu D., Christie H.D. Shear behavior of railway ballast based on large-scale triaxial tests. *Journal of Geotechnical and Geoenvironmental Engineering*. 1998;124(5):439-449. DOI: 10.1061/(asce)1090-0241(1998)124:5(439)

## Bionotes

**Vladimir I. Smirnov** — Dr. Sci. (Eng.), Associate Professor, Professor of the Department of “Mechanics and Strength of Materials and Structures”; **Emperor Alexander I St. Petersburg State Transport University (PGUPS)**; 9 Moskovsky pr., St. Petersburg, 190031, Russian Federation; ORCID: 0000-0001-5345-6165; vsmirnov1@gmail.com;

**Sergey A. Vidyushenkov** — Cand. Sci. (Eng.), Associate Professor, Head of the Department of “Mechanics and Strength of Materials and Structures”; **Emperor Alexander I St. Petersburg State Transport University (PGUPS)**; 9 Moskovsky pr., St. Petersburg, 190031, Russian Federation; ORCID: 0000-0002-5734-810X; baklava@mail.ru.

## Об авторах

**Владимир Игоревич Смирнов** — доктор технических наук, доцент, профессор кафедры «Механика и прочность материалов и конструкций»; **Петербургский государственный университет путей сообщения Императора Александра I (ПГУПС)**; 190031, г. Санкт-Петербург, Московский пр., д. 9; ORCID: 0000-0001-5345-6165; vsmirnov1@gmail.com;

**Сергей Александрович Видюшенков** — кандидат технических наук, доцент, заведующий кафедрой «Механика и прочность материалов и конструкций»; **Петербургский государственный университет путей сообщения Императора Александра I (ПГУПС)**; 190031, г. Санкт-Петербург, Московский пр., д. 9; ORCID: 0000-0002-5734-810X; baklava@mail.ru.

Contribution of the authors: the authors contributed equally to this article.

The authors declare no conflicts of interests.

Заявленный вклад авторов: все авторы сделали эквивалентный вклад в подготовку публикации.

Авторы заявляют об отсутствии конфликта интересов.

Corresponding author: Vladimir I. Smirnov, vsmirnov1@gmail.com.

Автор, ответственный за переписку: Владимир Игоревич Смирнов, vsmirnov1@gmail.com.

The article was submitted 25.03.2023; approved after reviewing 25.04.2023; accepted for publication 30.05.2023

Статья поступила в редакцию 25.03.2023; одобрена после рецензирования 25.04.2023; принята к публикации 30.05.2023.