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Feasibility study of calculation methods for tram track stiffness

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ABSTRACT Nowadays, design and construction of new urban tramlines require feasibility study of calculation methods for stiffness. A calculation method for tram track stiffness with noise and vibration insulation systems has been developed in this paper. Theoretical analysis of the impact of rail insulation systems on tram-to-slab load transfer has revealed the potential options and factors influencing the choice of a system. The new design method obtained makes it possible to predict the distribution of loads on the load-bearing foundation surface. Different variants of rail/slab load distribution have been determined depending on the mechanical characteristics of rail profiles. Tram track slab stiffness has been calculated for three design models such as rigid surface pavement, bridge structure and foundation slab. The design of the tram track as foundation slab allows calculating slab reinforcement as accurately as possible because the surface pavement takes into account the planned service life and tram flow density in the area under survey.

Field experiments have demonstrated a better convergence of theoretical and experimental data when designing the slab as foundation. As a result, a new method for calculating the foundation slab stiffness of a tram track has been proposed taking into account planned service life, tram flow density and other factors.

KEYWORDS: tram track; non-ballasted structure; design model; calculation method; track rail profiles; cyclic tests

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Научная статья

Выбор и обоснование методики расчета трамвайного пути на прочность

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АННОТАЦИЯ Строительство современных трамвайных линий и создание новых конструкций требуют обоснования методов расчета на прочность. Приведена методика расчета конструкции трамвайного пути на прочность при условии применения в ней систем шумо- и виброизоляции. Теоретический анализ влияния систем изоляции рельса на схему передачи нагрузки от трамвая на плиту показал возможные варианты и позволил выявить факторы, от которых эта схема зависит. Полученная новая расчетная схема дает возможность спрогнозировать распределение нагрузок на поверхность несущего основания. В зависимости от механических характеристик вкладышей определены варианты распределения нагрузки от рельса на плиту. Расчет плиты на прочность выполнен по трем схемам, как: дорожной одежды жесткого типа, мостовой конструкции и фундаментной плиты. Расчет трамвайной конструкции как фундаментной плиты дает возможность максимально точно учесть конструктив плиты (армирование), а расчет как дорожной одежды — учесть планируемый срок эксплуатации и интенсивность движения трамваев по исследуемому участку.

Полевые испытания продемонстрировали лучшую сходимость теоретических и опытных данных при расчете плиты как фундаментной. Благодаря анализу методов расчета предложили новую методику расчета несущей плиты трамвайного пути на прочность, которая учитывает планируемый срок эксплуатации, интенсивность движения трамваев и ряд других факторов.

КЛЮЧЕВЫЕ СЛОВА: трамвайный путь; безбалластная конструкция; расчетная схема; методика расчета; прирельсовые профили; циклические испытания

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INTRODUCTION

The calculation of tram track stiffness is based on the calculation of stiffness in its foundation components. There are two types of tram track design such as rail-sleeper structure and monolithic (concrete slab) structure. Design of tram tracks on sleepers is a standard procedure today as conventional methods are used for calculating a railway track [1]. The design of a monolithic slab as load-bearing foundation requires feasibility study of calculation methods as it is necessary to take into account the specific features of this type of design.

RAIL DYNAMICS CONSIDERATIONS IN RELATION WITH RAIL AND SLAB INSULATION SYSTEMS

The tram track slab is laid on a prepared sub-slab base layer consisting of sand, crushed stone and concrete. Depending on the conditions, the thickness of the structural layers varies and therefore the modulus of elasticity of the base layer on which the track slab is placed varies too. Another feature of modern tram track structures is the use of insulating materials such as rail profiles and/or vibration insulating mats (*Fig. 1*) [2–6].

According to the standard calculations of tram track stiffness, it is necessary to calculate the distribution of load from the tram to the load-bearing concrete slab. Theoretically, three options can be considered: point load from each wheel, load distributed over the section between the wheelset axles, and point load under the rail at the wheelset centre as it is the point of rail maximum deflection. Therefore, studies on the dependence of rail deflection on the coefficient of the plantar profile bedding and the tram axial load are important as they determine the rail/slab load transfer [7–15]. Rail deflection was calculated using formula (1).

$$y\left(x_n\right) = \sum_{j=1}^{N} \frac{P_j}{\beta^3 EI_z} e^{\left(-\beta\left|x_n - x_j\right|\right)} \times \times\left(\cos\left(-\beta\left|x_n - x_j\right|\right) + \sin\left(-\beta\left|x_n - x_j\right|\right)\right), \quad (1)$$

where x_n is the load application coordinate; N is the number of sections the rail is divided into; P_j is the tram axial load, kN; β is the slab relative stiffness modulus; x_j is the coordinate of the section under consideration; EI_z is the rail bending elasticity, Hm^2.

Calculations on rail deflection under different axial loads with different coefficients of the plantar profile bedding are shown in *Fig. 2 and 3*.

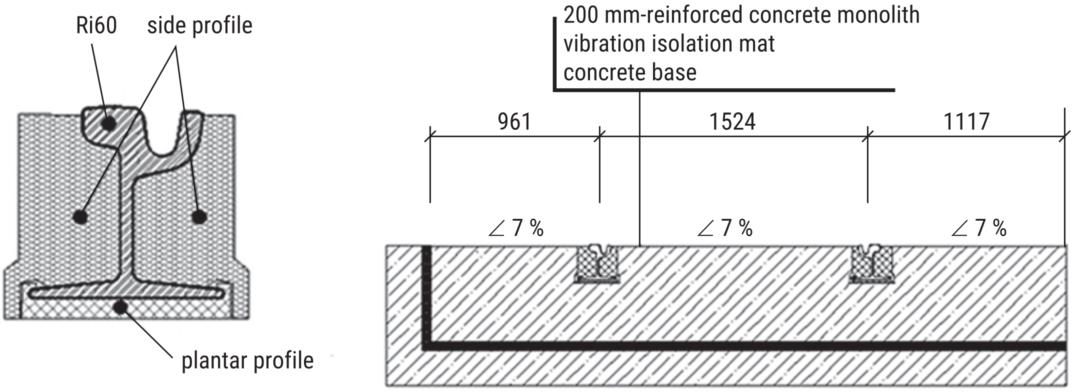

Fig. 1. Application of insulating materials

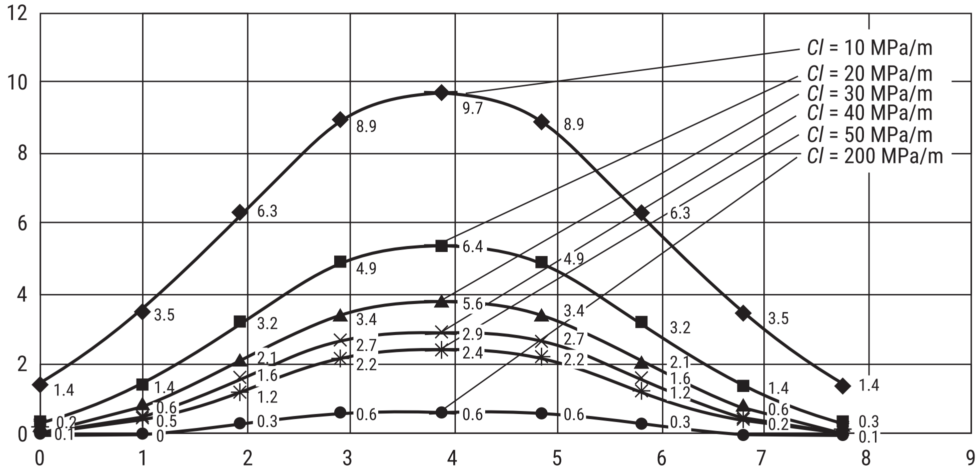

Fig. 2. Rail deflection (mm) depending on the coefficient of the plantar profile bedding C_1 (MPa/m) at P_{os} = 83.3 kN/axis in different sections

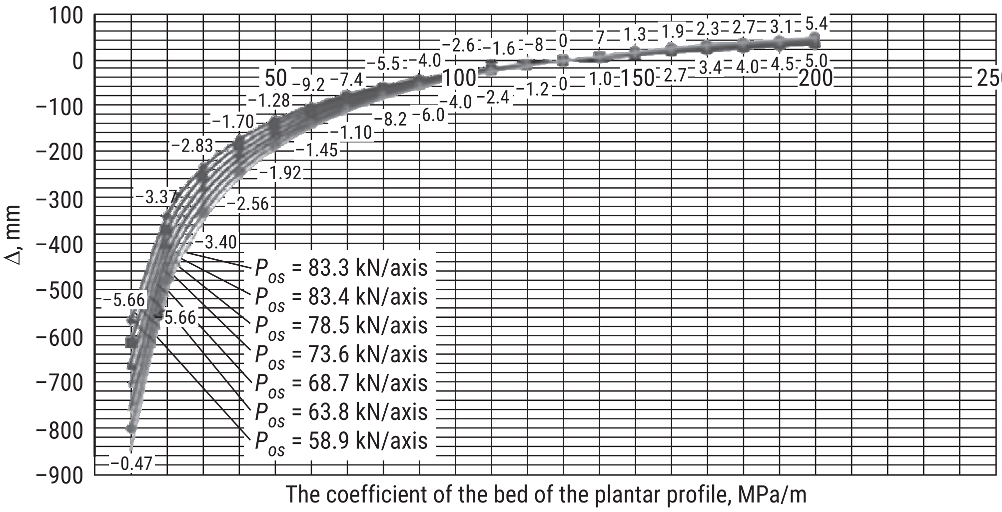

Fig. 3. Rail deflection $\Delta = \Delta_{abs} = h_k - h_s$ ($mm \cdot 10^{-3}$) under different axial loads Pos (kN/axis) depending on the coefficient of the plantar profile bedding C_1 (MPa/m), where h_k is rail deflection under tram car wheels; h_s is rail deflection at the midpoint between wheel pairs

The studies have shown that the axial load does not effect the rail deflection shape. This depends mainly on the coefficient of the plantar profile bedding. Theoretically, we can assume three types of load transfer from the tram bogie through the rail to the concrete slab surface depending on the stiffness of the plantar profile (*Fig. 4–6*). They are point load (separately from each wheel), load distributed in the section between the wheelset axles, and point load from the rail in the centre between the wheels, given rail maximum deflection in that section (*Fig. 2*) [7–11].

A tram track on a monolithic foundation can be calculated according to three types of design such as rigid surface pavement, bridge structure and foundation slab [3, 6, 7]. According to these three types, the calculations were made taking into account the specifics of rail/slab load transfer.

The analysis of the results showed that the calculation of the tram track-bearing slab as a rigid surface pavement is limited to the determination of stiffness factor K_{pr}. It is calculated by the formula

$$K_{pr} = 1{,}0 < \frac{R_{ri}^{rasch}}{\sigma_{pt}}, \tag{2}$$

where R_{pt}^{rasch} is calculated concrete tension stiffness at bending, σ_{pt} is tensile stresses at bending caused by load action on the concrete slab taking into account the temperature difference across the thickness of the slab.

This method of calculation does not allow determining the distribution of forces arising in the slab in sections and does not take into account the nature of reinforcement. As a result, we obtain stresses only at the points of rail/concrete surface contact.

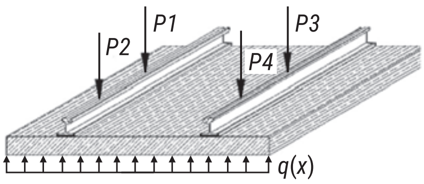

Fig. 4. Load transfer to the bearing concrete slab with plantar profile stiffness over 130 MPa/m

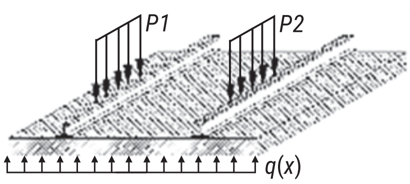

Fig. 5. Load transfer to the load-bearing concrete slab with the planar profile stiffness of 130 MPa/m

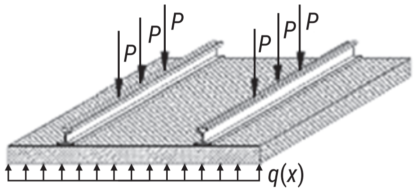

Fig. 6. Load transfer to the bearing concrete slab with the planar profile stiffness from 10 to 130 MPa/m

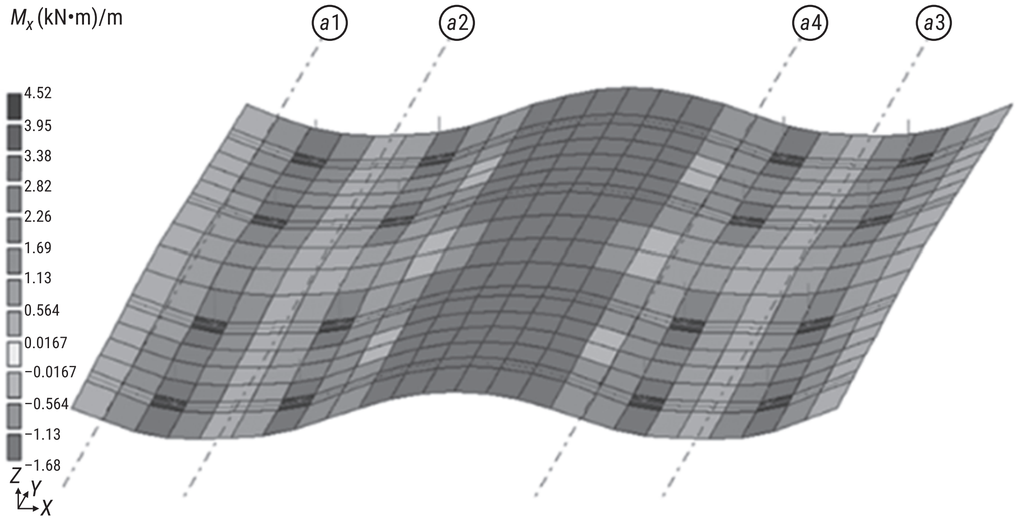

Fig. 7. Mosaic of stresses due to bending moment M_x (kN/m) with axial load of 58.86 kN/axis and coefficient of planar profile bedding C_1 = 50,000 kN/m

The design of the tram track stiffness as a bridge structure using a simplified calculation scheme (beam on two supports) is unacceptable. The calculations are reduced to the slab with elastic foundation that is the foundation slab without taking into account the base layer performance under the slab [12–19].

When the tram track is designed as foundation slab, the forces occurring in each final element are determined (*Fig. 7*), thus, allowing us to analyse the overall stress state of the slab with different variants of its reinforcement.

To verify the results of theoretical calculations, laboratory and field tests were carried out. For laboratory tests, a tram track structural element was made and it was subjected to cyclic loads. Thus, the dependence of the stresses caused in the track structural elements on its service life was obtained [11, 15, 19]. The test bench is shown in *Fig. 8*.

The results are presented in *the Table*.

The analysis of the results obtained confirms that the qualitative picture of the stress distribution in the slab obtained in the theoretical and laboratory studies is similar. As the number of cycles increased, the stiffness characteristics of the rail profiles were modified, which should be taken into account when developing the design model.

Field tests were carried out using strain gauges to measure the stresses in the concrete slab as the tram passed (*Fig. 9*).

The experimental studies have shown that the calculation data of the tram track as a foundation slab have the greatest convergence with the experimental data (the difference between the experimental and theoretical results was 16 %) [2, 5].

This method can be recommended for designing a tram track on a concrete slab. It provides for the cal-

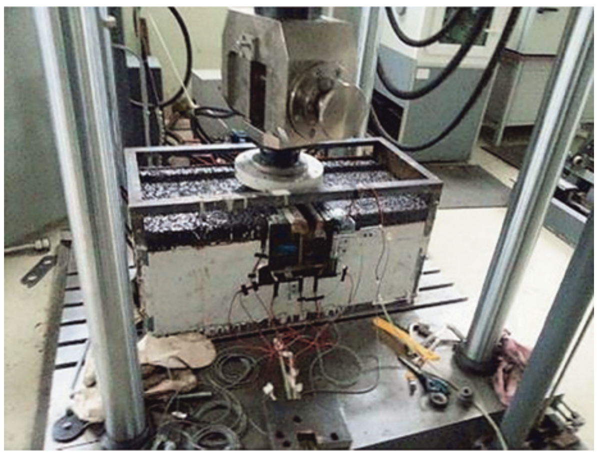

Fig. 8. A tram track structural element used in cyclic tests

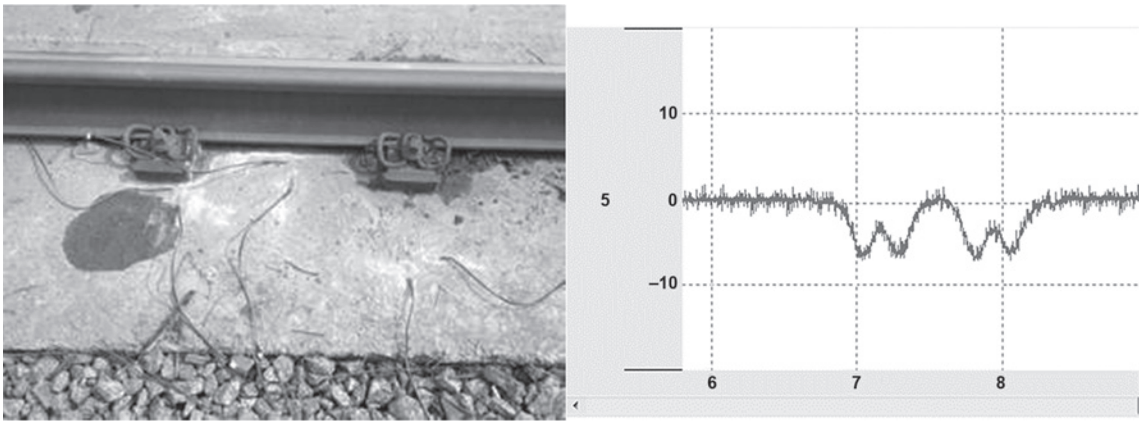

Fig. 9. Example of recording the bending stresses σ_x (kg/cm^2) in the slab during the passage of an empty tramcar

Table

Cycles completed, million	Stresses, kgf/cm^2							
	Concrete under rail foot (compression in all areas)							Rubber "at the lip", horizontal (tension)
	Below "left" edge, horizontal	Below "left" edge, vertical	In the rail axis, horizontal	In the rail axis, vertical	Below "right" edge, horizontal	Below "right" edge, vertical	Safety margin, %	
0	–(7–8)	–(13–15)	–(31–33)	–(7–8)	–(8–10)	–(7–8)	87	+(1–1,2)
3,37	–(8–10)	–(10–15)	–(34–38)	–(7,5–9,5)	–(9–12)	–(7,5–9)	86	+(0,7–0,9)
6,755	–(7,5–9)	–(9–12)	–(33–39)	–(7–8)	–(7–10)	–(7–9,5)	85	+(0,7–0,9)
9,94	–(8–9)	–(12–14)	–(34–39)	–(9–10,5)	–(9–11,5)	–(11–12)	85	+(0,7–0,9)

culation of the tram track structure in two stages: the determination of the sub-slab layer bedding coefficient and the determination of the bending forces generated in the load-bearing slab. At the same time, the design of the tram track as rigid surface pavement makes it possible to take into account a number of factors that influence the stiffness of the structure, such as traffic density, duration of load application, etc., by introducing relevant factors which are not taken into account in the design as foundation slab. It is therefore necessary to synthesize these two calculation schemes. As a result, a formula is proposed for calculating the stiffness of the tram track load-bearing slab that takes into account all the factors mentioned above.

$$R_{bt,f} = \sigma_{ri,f} \cdot k_y \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4,$$

where $R_{bt,f}$ is the design tensile stiffness of concrete at bending; $\sigma_{ri,f}$ is the standard tensile stiffness of polymer-fibrated concrete at bending; k_y is the fatigue factor of concrete under repeated loads. It is determined by the formula

$$K_y = 1{,}08(\Sigma N_p)^{-0{,}063},$$

where ΣN_p is the total number of applications of the given load during the calculated lifetime; k_1 is the coefficient of concrete behaviour which takes into account duration of the load action; k_2 is the coefficient of concrete behaviour which takes into account the concrete layer height; k_3 is the coefficient of concrete behaviour which takes into account the alternate freezing and thawing of concrete; and k_4 is the coefficient of concrete behaviour which takes into account the character of concrete structure failure.

CONCLUSION

The calculation of the stiffness of a tram track load-bearing slab as foundation shows the most reliable results as the difference between theoretical and experimental data is 16 %. This method makes it possible to determine the stresses caused in each element taking into account the nature of reinforcement. This allows analyzing the overall stress state of the slab. The improved method mentioned above (taking into account additional factors) is recommended for calculation of the tram track load-bearing slab for stiffness.

The linear dependence of occurring bending stresses on the sub-slab layer stiffness and axial loads within given tram axial loads and bedding coefficient allows for the unification of consolidated tables of values occurring in the tram track load-bearing slab depending on the bedding coefficient and tram axial load for a given slab geometry. Thereby simplifying the calculations for the slab stiffness.

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